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COMBINED FORCED AND FREE CONVECTION FOR LAMINAR FLOW IN HORIZONTAL TUBES WITH UNIFORM HEAT FLUX

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Abstract—This study considers the effects of free convection on laminar flow of water in horizontal circular tubes having essentially constant heat flux at the tube wall. A visual and quantitative study was performed utilizing electrically heated glass tubing. These data were combined with other data and correlations to obtain a general picture of the influence of free convection on the Nusselt number. The final correlation curves given in Fig. 11 are provisionally recommended for obtaining heat–transfer coefficients in practical situations. With reasonable heating rates, the heat–transfer coefficients can be three to four times the values predicted by traditional constant property solutions.

NOMENCLATURE

c_p, specific heat at constant pressure;
D, inside diameter of tube;
G, mass velocity;
g, gravitational acceleration;
h, heat–transfer coefficient;
k, thermal conductivity;
L, heated length of test section;
q'', heat flux;
T, temperature;
ΔT, temperature dfference ($= T_w - T_b$);
x, axial coordinate along heated length;
β, bulk modulus of expansion;
μ, dynamic viscosity;
ν, kinematic viscosity.

Subscripts
b, denotes bulk or mixed–mean fluid condition;
CP, denotes solution assuming constant properties;
tr, denotes onset of significant free convection effects;
w, denotes condition at inside wall of tube.

Dimensionless groups
Nu, Nusselt number ($= hD/k$)–circumferential average;
Pr, Prandtl number ($= c_p\mu/k$);
Ra, Rayleigh number $\left(= \dfrac{g\beta\Delta T D^3}{\nu^2 k}c_p\mu\right)$ —circumferential average;
Re, Reynolds number ($= GD/\mu$).

All properties are evaluated at the local bulk fluid temperature unless otherwise noted.

INTRODUCTION

LAMINAR flow heat transfer in tubes is encountered in a wide variety of engineering situations. The prediction of laminar heat-transfer coefficients is generally accomplished by referring to current textbooks, which are well stocked with equations and graphs based on analytical solutions to the governing equations for laminar flow. These analyses are generally restricted to constant fluid properties. In actual practice, however, experimental data exhibit substantial deviations from the analytical predictions, in large part due to inadequacy of the constant property assumption. Accordingly, recent analytical and experimental work has focussed on determining the effects of variable properties. The studies generally divide into those which consider variable transport properties, particularly viscosity, and those which consider variable density. A distinction can easily be made in an analytical formulation; however, it is clearly difficult to sort out the two effects in an experiment. At present, the variable property work has been applied to only a fraction of the geometeries and boundary conditions for which the constant property solutions are available. This is understandable since the results must be obtained for a specific working fluid or flow orientation.

The present paper is concerned with the problem of laminar flow in horizontal circular tubes having a uniform heat flux boundary condition. With appreciable heating, the fluid near the wall rises and a secondary fluid motion is established which is symmetrical about a vertical plane passing through the tube axis. For this boundary condition, a wall–minus–fluid temperature difference obtains throughout the length of the heated tube; thus the free convection persists throughout the tube. For sufficiently long tubes a "fully developed" condition can be reached, where the heat–transfer coefficient changes only due to changes in the temperature level of the fluid. This combined forced and free convection results in a heat-transfer coefficient well above that predicted by the constant property analysis.

An analysis of the fully developed case with water, assuming density to be the only temperature–dependent property, was recently reported [1, 2]. Using finite–difference techniques and a large-scale digital computer, solutions were obtained for two limiting boundary conditions: zero circumferential heat conduction in the tube wall ("Glass tube") and uniform temperature around the tube circumference ("Infinite conductivity tube"). Neglect of the temperature dependence of the transport properties was not considered to be serious in the case of water. In any event, limitations on computer storage and running time precluded solution of either the variable transport property case or the three-dimensional developing case.

It should be noted at this point that the finite–difference approach appears to be the only method which will yield accurate predictions over a wide range of conditions. Hanratty [3], Morton [4], Iqbal and Stachiewicz [5, 6], and Faris and Viskanta [7] have developed series solutions for the case of fully developed flow, generally using Rayleigh number as a perturbation parameter. However, these solutions are accurate only for small values of the expansion parameters, and give unrealistically high estimates of Nusselt number at the higher Rayleigh numbers of practical interest. An early boundary layer solution by Mikesell [8] was unsuccessful; however, a recent boundary layer analyses by Mikesell and co-workers [9] agreed well with data for ethylene glycol at large Rayleigh number. The integral boundary layer solution of Mori and Futagmi [10], which is restricted to a Prandtl number of about unity, agreed well with limited air data reported by Mori *et al.* [11]. Siegwarth and Hanratty [12] recently developed a finite–difference solution and obtained detailed flow and temperature profiles for several cases with ethylene glycol.

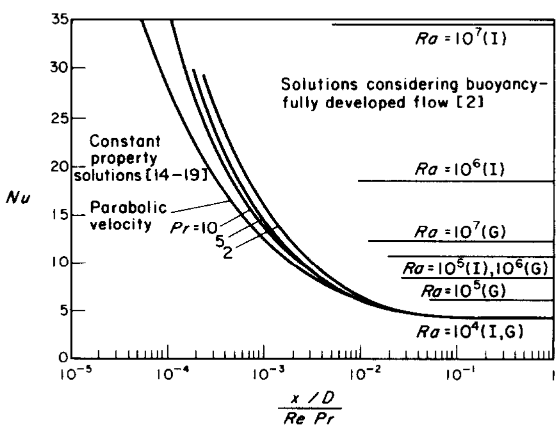

FIG. 1. Analytical solutions for laminar flow heat transfer in circular tubes.

The current analytical status of this problem is summarized in Fig. 1 where the developing constant property solutions are combined with the limiting solutions for the fully developed secondary flow case. The constant property solution for a developed parabolic velocity profile is a composite of the analytical solutions given in [13–16] which agree quite closely even though various techniques were used. The solutions for developing velocity and temperature profiles ($Pr = 2, 5, 10$) were taken from the solutions given in [15, 17, 18] which also agree quite well. The analytical solutions for secondary flow were taken from a curve fit to the computed data points for water (heating and cooling) given in [2], considering Rayleigh number evaluated at the bulk temperature. The complete curves for these asymptotes are provided in Figs. 5 and 10.

Figure 1 appears to be of limited use in predicting laminar heat-transfer coefficients in the presence of significant secondary flow effects, since the computer solutions have not been obtained for the developing region. Furthermore, since the upper and lower bounds (I and G) differ greatly at high Grashof number, solutions would also be required for a range of tube thermal conductivities and wall thicknesses. These analytical extensions would entail an extremely long computer production; hence, it seems appropriate at this time to see if experimental data can provide the necessary information to convert Fig. 1 to a reliable design chart.

Experiments were recently performed with electrically heated glass tubes in an attempt to coroborate the analytical lower bound for fully developed secondary flow. The present paper describes this experimental work and outlines the development of a complete correlation plot for water based on all available experimental information.

EXPERIMENTAL PROGRAM

Test section

The basic test section consisted of a 30-in. length of 0·433-in. i.d. Pyrex E–C Coated Tube. This tubing represented the closest possible approximation to the boundary conditions used in the analytical model (G), i.e. constant heat flux and zero heat conduction around the tube circumference. Since the tubing is 70 per cent optically transparent, it was also possible to carry out a visual study of the secondary flow development. As shown in Fig. 2 the tube was

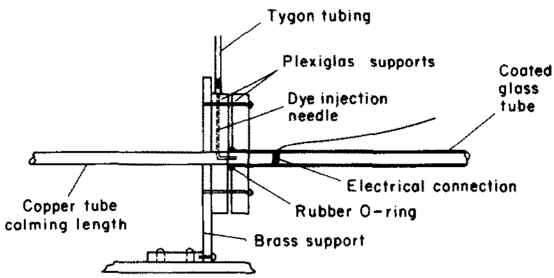

FIG. 2. Schematic of basic test section.

supported at either end in a frame made from Plexiglas and brass. One Plexiglas support plate was bored to the o.d. of the glass tubing and the other to the i.d. of the tubing. A rubber O-ring between the plates provided sealing when the plates were bolted to the brass support. A dye injection needle made from 0·028-in. o.d. hypodermic tubing was mounted in the middle support plate so that the tip was at the tube centerline. This needle was installed only for visual tests. A 36-in. length of 0·43-in. i.d. copper tubing was used for the entrance section. A thermocouple to measure the inlet liquid temperature was mounted in the calming length, and an exit thermocouple was installed in a mixing chamber. The entire test-section assembly was heavily insulated with glass fiber insulation, except during visual tests.

Four wall thermocouples were placed circumferentially 90 degrees apart, at the same axial position, to record the outer tube wall temperature profile. Due to electrical pick-up it was necessary to electrically insulate the thermocouple beads from the wall with thin strips of mica. A guard shield was then required to insure that the thermocouples were located in an adiabatic region. In anticipation of a rather large circumferential variation in tube wall temperature, a guard shield was constructed which could be heated to approximately match this temperature variation. The guard shield shown in Fig. 3 was constructed from aluminium pipe, 2 in .i.d. and 6 in. long, longitudinally slit nearly through to provide four separate sections. After insulation, each section was wrapped with nichrome heater wire connected to a power supply. Thermocouples were attached to the inside surface of the guard heater opposite the corresponding tube wall thermocouple. The

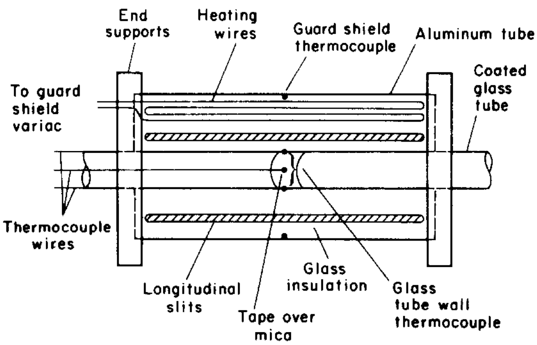

FIG. 3. Guard shield and wall thermocouple installations.

assembly was completed by filling the heater tube with glass fiber insulation and leading the eight thermocouples out through the Micarta end supports.

Procedure

The test-section assembly was connected to a low pressure loop circulating demineralized and degassed distilled water. Through use of an accumulator and appropriate valving, pressure fluctuations at the test section were reduced to a negligible value. During the visual tests, the circulating system was inoperative and relatively gas-free city water was directed through the flowmeter and test section before entering a drain. A dilute solution of potassium permanganate was supplied to the injection needles from an elevated reservoir.

The testing generally proceeded by increasing the a.c. test-section power at constant flow rate and inlet temperature. The guard heater was kept in a fixed position near the end of the glass tube, and various heated lengths were achieved by moving the upstream electrical connection. Variacs were used to control the power to the various guard heater segments so that the heater temperatures were within several degrees of the adjacent tube temperatures. Readings were corrected for the temperature drop through the glass wall. With measured temperatures at 0, 90 and 180 degrees and zero temperature gradient at 0 and 180 degrees, it was possible to draw an accurate curve to represent the temperature profile around the inner tube circumference. Representative wall temperature data are given in Fig. 4. A planimeter was used to obtain the average tube wall temperature. The heat flux was obtained from the average power measurement and inside

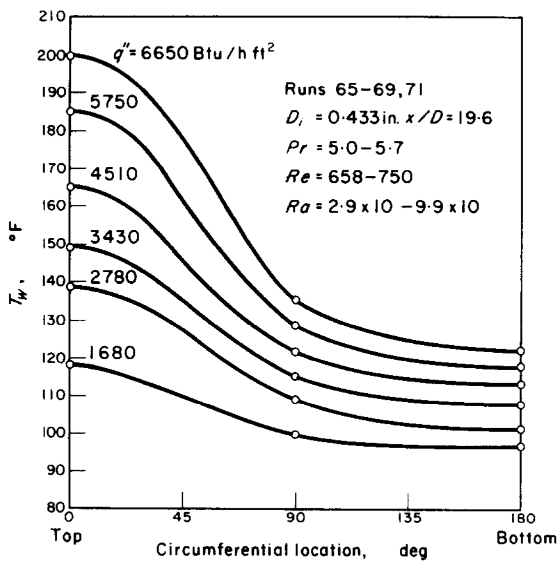

FIG. 4. Typical inner wall temperature profiles.

tube area, a procedure which was felt to be justified in view of the small temperature coefficient of resistivity for the tube coating. The local bulk fluid temperature was computed from flow rate, power, and inlet temperature measurements. These quantities sufficed for evaluation of the conventional dimensionless groups used for data representation. Further details of the test procedure and data reduction are given in [19].

DISCUSSION OF TEST RESULTS

Visual studies

A 35-mm camera with closeup lens was mounted on tracks parallel to the glass tube to permit photographing top and side views. Color pictures were taken using photoflood lamps for illumination. By combining prints for a sequence of snapshots taken along the tube, it was possible to prepare composite photographs of the developing flow. These pictures of dye-identified streamlines cannot be reproduced here; however, it is sufficient to note the pertinent qualitative observations. The dye clearly delineated the spiraling streamlines characteristic of developing secondary flow. Raising the heat flux at constant flow rate tended to decrease the axial pitch of the streamlines, while the same effect was produced by decreasing the flow at constant heat flux. No backflow was observed during any of the tests in accordance with the prediction of Mikesell [8] which is given as $Nu\,Gr/Pr\,Re^2 > 150$; the value of this parameter was a maximum of about 20 for these tests.

The dye trajectories were used a a crude test of fully developed flow. Since the axial pitch for particles injected at the tube axis is very large compared to those injected near the eye of a fully developed streamline [2], it seemed reasonable to conclude that a fully developed condition occurred when the dye completed at least one spiral by the time it reached the end of the tube. This condition was satisfied for a considerable range of heat fluxes and flow rates.

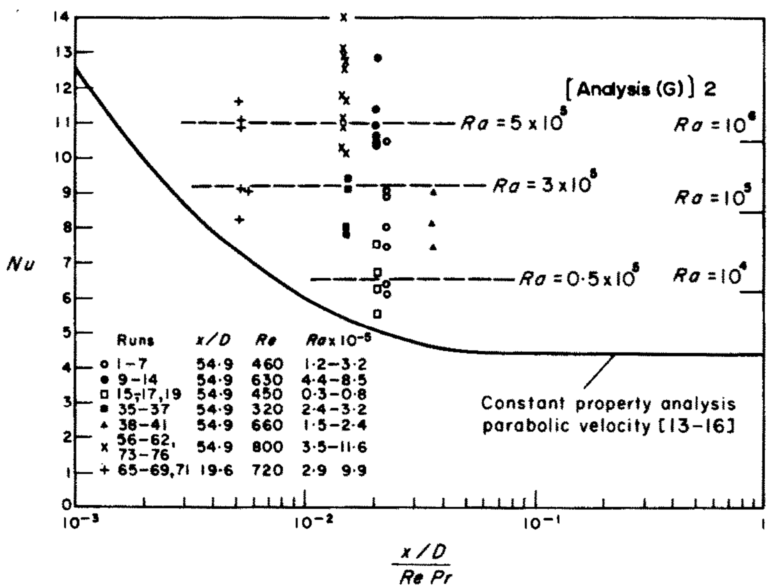

FIG. 5. Average heat transfer data for laminar flow in a horizontal glass tube with uniform heat flux.

Heat-transfer results

A typical set of wall temperature profiles is given in Fig. 4. The temperature difference between top and bottom of the tube increases as the heat flux is increased due to the increasing stratification of the flow. The glass tube clearly supports a large circumferential temperature gradient. It was found, however, that the temperature difference between top and bottom generally was somewhat lower than that predicted by the correlation of analytical results given in [2].

The average heat-transfer coefficients are plotted according to the usual coordinates in Fig. 5. The vertical spread in each set of data represents, operationally, a range of heat fluxes, with the highest Nusselt numbers generally corresponding to the highest heat fluxes. Since the Rayleigh number was a dependent variable, lines of constant Ra could only be obtained by interpolation or extrapolation of the data. Typical lines of constant Ra are indicated; these lines are essentially horizontal for the range of $(x/D)/Re\,Pr$ considered. This suggests that the data were taken under conditions where the flow is fully developed, as was qualitatively indicated by the visual observations. When the experimental data for constant Ra are compared with the analytical glass tube results, it is seen that there is a systematic deviation of the data from the analysis.

Assuming that the data represent fully developed conditions, a more detailed comparison can be made as is shown in Fig. 6. The

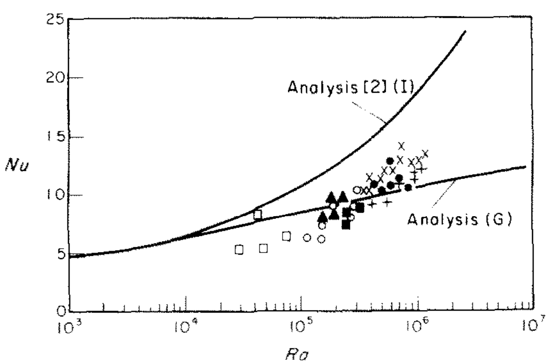

FIG. 6. Comparison of present data with analytical solutions for fully developed flow.

systematic deviation noted in Fig. 5 is quite evident in this figure, with the data lying generally below the prediction (G) for $Ra < 4 \times 10^5$ and above the prediction for higher values of Ra. A partial explanation for this behavior can be given by considering the experimental conditions.

It is noted first of all that the data exhibit considerable scatter. The experimental uncertainty in Nu is of the order of 10 per cent when probable uncertainty in wall temperature measurement (guard heater adjustment and thermocouple error), determination of the average wall temperature, bulk temperature, and heat flux are taken into account. The uncertainity is especially large for $Ra < 10^5$ where ΔT is less than 5°F. With regard to the heat flux, one of the inherent problems with this type of tubing is that the conductive coating tends to be nonuniform, both axially and circumferentially. The extent of the non-uniformity is somewhat speculative, however, since it is difficult to gauge the 16-μin. coating.

In general, it is recognized that the test tubing is not quite faithful to the boundary conditions of the analysis. Pyrex has a thermal conductivity about twice that of water; hence, some circumferential conduction is to be expected. Furthermore, although the temperature coefficient of resistivity is small ($< 0{\cdot}001$/°F from test data), it is still large enough to cause some circumferential current redistribution when the temperature gradients are large. Both of these factors would produce greater heat-transfer coefficients than if the ideal boundary conditions were met, since the secondary flow is increased by redistributing the wall heat flux toward the lower portion of the tube.

A further consideration in these experiments is variable transport properties. A viscosity ratio factor, $(\mu_w/\mu_b)^{0{\cdot}14}$, suggested by Sieder and Tate [20], is generally utilized for correcting data in order to make a comparison with constant property predictions. Modifications of the exponent in this factor, for low $(x/D)/Re\,Pr$ have recently been proposed by Shannon and Depew [21]. Since the correction in the present case reduces the Nusselt number by a maximum of 5 per cent, it was not applied to the data. There is still a possibility, however, that the variable transport property effects and buoyancy effects would interact to give a more significant correction.

In any event, the present data appear to be reliable enough to consider them for use in formulating a general correlation of free convection effects, which is a major goal of this work.

REVIEW OF AVAILABLE DATA

Ede [22] reported one of the earliest studies of the effects of free convection on laminar flow where the uniform heat flux boundary conditions was applied. Aluminium–brass pipes, with diameters ranging from 0·5–2·0 in. and wall thicknesses up to 0·279 in., were electrically heated (d.c.). Five insulated thermocouples were uniformly spaced around the circumference at each axial position. Details of the inlet geometry were not given. The correlation, which is primarily based on data for fully developed flow of water, is given as

$$Nu = 4{\cdot}36(1 + 0{\cdot}06\,Gr^{0{\cdot}3}). \qquad (1)$$

Using the cited average value of $Pr = 8$ for the water tests, this becomes

$$Nu = 4{\cdot}36(1 + 0{\cdot}0321\,Ra^{0{\cdot}3}). \qquad (2)$$

Roy [15] took a large amount of data with an electrically heated (a.c.) 304 stainless tube, 0·62 in. i.d. with 0·065 in. wall. The water entered the tube through a bellmouth nozzle. Thermocouples were installed along the length of the tube, but only at the top of the tube. Since the tube could support a substantial circumferential temperature gradient, this thermocouple would expected to give a conservative indication of the average heat-transfer coefficient when there is appreciable secondary flow. Data for those typical tests are given in Fig. 7. The data at low $(x/D)/Re\,Pr$ lie below the constant property solution for approximately the same Prandtl number, suggesting that some stratification may be occurring near the tube inlet. Certainly the downstream results are not a reliable indication of the heat-transfer coefficient for the fully developed flow. On the other hand, the data should give a reasonable indication of the onset of first significant secondary flow.

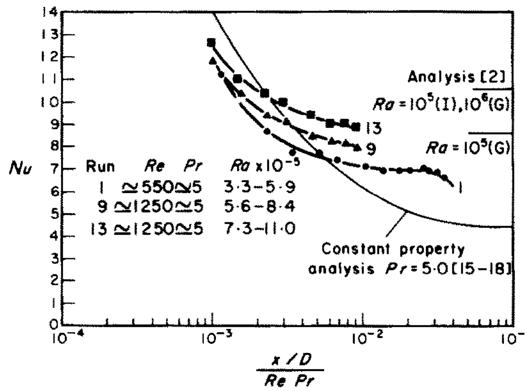

FIG. 7. Representative heat transfer data of Roy [15].

There appear to be no other data which illustrate secondary flow effects when both hydrodynamic and thermal boundary layers are developing.

Petukhov *et al.* [23–25] conducted a comprehensive investigation of laminar flow of water in an electrically heated (a.c.) tube. The stainless steel tube was 0·743-in. i.d. with 0·014-in. wall; the heated length of 73-in. was preceded by a 71-in. calming length. Numerous thermocouples were attached to the tube wall at various axial and circumferential locations. The tube was also rotated to provide greater refinement of the circumferential temperature distribution. A simple analysis of conduction around the tube wall was made so that more accurate values of the local heat flux and heat-transfer coefficient could be obtained; of primary interest to the present study are a selected composite of average Nusselt numbers (based on average heat flux and wall temperature) plotted in [23]. These data are shown in Fig. 8, plotted in a format similar to that used in Figs. 5 and 7. The asymptotic values of the Nusselt number for each data set represent a constant Rayleigh number corresponding to the upper value indicated. These data clearly indicate the

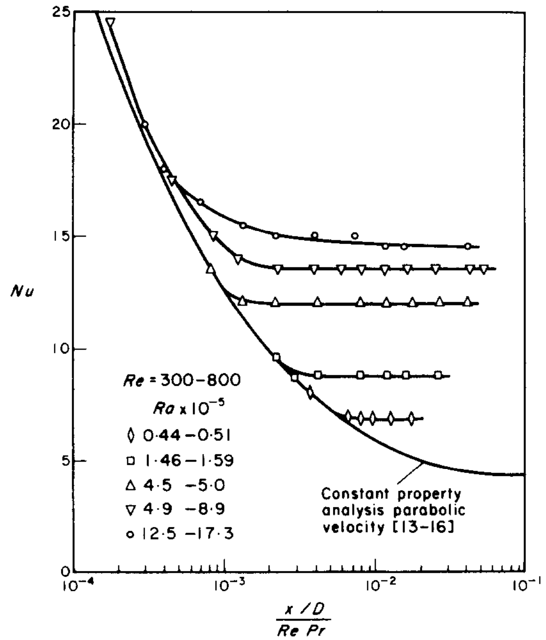

FIG. 8. Average heat transfer data of Petukhov and Polyakov [23].

main features of the combined free and forced convection. The departure from the constant property prediction occurs at a rather well-defined combination of Ra and $(x/D)/Re\,Pr$. A relatively short transition region occurs before the constant Nu corresponding to fully developed secondary flow is reached. The developing length for secondary flow is clearly much shorter than that required in the absence of free convection effects.

Petukhov and Polyakov [23] also suggested a transition criterion for the onset of significant free convection effects as

$$Ra_{tr} = \frac{5 \times 10^3}{\left(\frac{x/D}{Re\,Pr}\right) Nu} \text{ for } \frac{x/D}{Re\,Pr} < 1{\cdot}7 \times 10^{-3} \quad (3)$$

$$Ra_{tr} = 1{\cdot}8 \times 10^4 + \frac{55}{\left(\frac{x/D}{Re\,Pr}\right)^{1{\cdot}7}} \quad \text{for } \frac{x/D}{Re\,Pr} > 1{\cdot}7 \times 10^{-3}. \quad (4)$$

Their correlation for the asymptotic Nusselt number in fully developed free convection is

$$Nu = 4{\cdot}36 \left[1 + \left(\frac{Nu\,Pr}{1{\cdot}8 \times 10^4}\right)\right]^{0{\cdot}045} \quad (5)$$

Shannon and Depew [26] studied free convection effects in an electrically heated (d.c.) stainless tube with 0·305-in. i.d., 0·035-in. wall, length of 240 in., and calming section of 40 in. The average temperature at various axial locations was monitored by thermocouples soldered to a copper strap around the tube circumference which was insulated from the tube wall by a thin layer of tape. Water was introduced into the test section at 32°F, and a range of heat fluxes was applied to demonstrate the effects of superimposed free convection. Rayleigh numbers up to about 10^6 were obtained. The data exhibit characteristics similar to those seen in Figs. 7 and 8; however the trends are less clear since large variations in Ra occurred along the length of the tube due to the large increase in fluid temperature. Since it is difficult to transcribe and compute the necessary parameters from the published data, similar plots were not prepared. However, use will be made of the general correlation developed by these investigators. This correlation was presented as a plot of $Nu - Nu_{CP}$ vs. $Ra^{\frac{1}{4}}/Nu_{CP}$. The criterion

$$Ra^{\frac{1}{4}}/Nu_{CP} = 2 \quad (6)$$

was suggested as describing the onset of significant free convection effects.

The data described in this review will now be used together with the present data to define the probable behavior of a system where appreciable free convection effects are present.

DEVELOPMENT OF A CORRELATION PLOT

Transition region

The investigations of Petukhov and Polyakov [23] and Shannon and Depew [26] provide data for the onset of secondary flow effects in a system having developed hydrodynamic conditions at the inlet of the heated tube. The suggested correlations given by equations (3), (4) and (6) are indicated in the transition plot of Fig. 9. The data of Petukhov and Polyakov as shown in Fig. 8 are included for comparison. In general there appears to be rather good agreement between the two correlations and data at higher $(x/D)/Re\,Pr$. For lower values of this parameter, equation (3) diverges from both the data and the correlation of equation (6). This comparison suggests that equation (4) might, in fact, be more suitable for defining transition throughout the entire range.

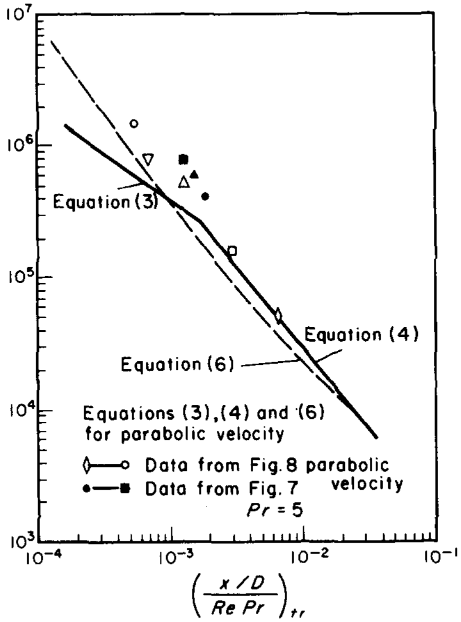

FIG. 9. Data and correlations for onset of significant free convection effects.

The data of Roy [15] provide the only available information for transitions with developing hydrodynamic and thermal boundary layers. although the data are somewhat difficult to interpret due to inadequate thermocouples, a reasonable estimate of transitions can be obtained from the information in Fig. 7 and tabulations in his thesis. As indicated in Fig· 9, the transitions are displaced to higher $(x/D)/Re Pr$, as would be expected.

Fully developed region

A composite of available data for fully developed secondary flow is given in Fig. 10. The best-fit curve for the present glass tube data is included with the correlations suggested for metal tubes [22, 23, 26]. Considering the diversity of the experiments, there is surprisingly close agreement between these correlations. This suggests that a composite fit of the various expressions can be made to provide an effective

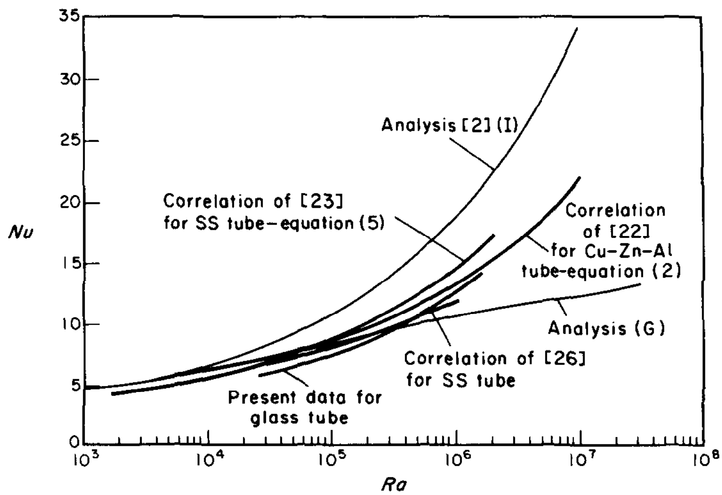

FIG. 10. Composite of correlations for fully developed secondary flow in glass and metal tubes.

general correlation which should be valid for most practical situations.

With regard to the analytical predictions, it is evident that the infinite conductivity tube solution represents a clear upper bound to the average Nusselt number. While the glass tube solution is somewhat optimistic at lower Rayleigh numbers, it does give a definite lower bound for the higher Ra where there is a very strong influence of free convection.

The composite prediction plot

The traditional constant property solutions given in Fig. 1 were combined with the data for superimposed forced and free convection shown in Figs. 9 and 10 to give the final prediction plot of Fig. 11. The fully developed asymptotes for a range of Rayleigh numbers were rather well established by fitting a mean curve to the various correlations plotted in Fig. 10. The onset of significant free convection could be accurately defined only for the use of hydrodynamically developed flow at the tube inlet. However, the limited data of Roy for $Pr = 5$ gave a reasonable indication of what to expect in the developing region for $Ra \sim 5 \times 10^5$. A consistent set of transition curves was faired-in for each value of Ra and Pr. The transition region generally occupies a rather small portion of the curve for each Pr–Ra combination. From a practical point of view, then, there is little incentive for undertaking an analytical solution of the developing secondary flow problem.

Figure 11 thus represents the probable behavior of the Nusselt number with laminar flow of water in a horizontal circular tube with a constant wall heat flux. This figure is then the provisionally recommended correlation plot. In general, any reasonable heat flux will give rise to appreciable free convection, with the result that the average wall temperature will be below that predicted by the traditional constant property analyses. With regard to the upper limit of Reynolds number for which these data apply, Petukhov *et al.* [25] have reported that the transition to turbulent flow takes place at higher Reynolds numbers as Rayleigh number is increased.

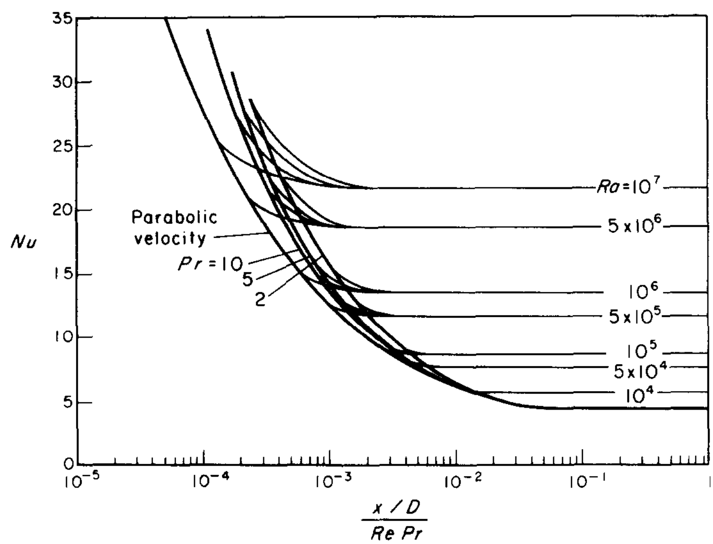

FIG. 11. Predictions plot for laminar flow of water in a horizontal circular tube with constant heat flux.

CONCLUDING REMARKS

This study has elucidated the effects of free convection on laminar flow on water in horizontal circular tubes having essentially constant heat flux at the tube wall. The experimental investigation demonstrated that electrically heated glass tubing is advantageous for testing since both visual flow pattern observations and quantitative information can be obtained. The glass tube provided the closest possible approximation to the boundary conditions used in a previous analysis; however, conduction around the tube circumference and nonuniform heat generation introduced experimental uncertainties.

The present data were combined with reported data and correlations to obtain a general picture of the influence of secondary flow on the Nusselt number for water. A review of the reported data indicates that the developing length for secondary flow is shorter than that required in the absence of free convection effects. Furthermore, the developing region is rather short. There is thus ample justification for restricting analytical solutions to the more tractible case of fully established secondary flow. Strictly speaking, there is no such thing as a general correlation of the problem since the tube geometry and material cannot be included in simple terms. However, the available data for various tube sizes and materials fall into a sufficiently narrow band so that these factors can be neglected. The final correlation plot of Fig. 11 should give an accurate prediction of the heat-transfer coefficient in any practical situation. In general, any reasonable heating rate will give rise to large departures from the traditional constant property solutions; for example, at a moderate value of $Ra = 10^6$, Nu is over three times the constant property value for developed flow.

ACKNOWLEDGEMENTS

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CONVECTION MIXTE POUR UN ECOULEMENT LAMINAIRE DANS DES TUBES HORIZONTAUX AVEC UN FLUX THERMIQUE UNIFORME

Résumé—Cette étude concerne les effets de la convection naturelle sur l'écoulement laminaire d'eau dans des tubes horizontaux à section circulaire ayant un flux thermique pariétal constant. On conduit une étude quantitative par visualisation à l'aide de tube en verre chauffés électriquement. Ces mesures sont combinées avec d'autres pour obtenir un tableau général de l'influence de la convection naturelle sur le nombre de Nusselt. Les courbes finales corrélatives données Fig. 11 sont recommandées pour obtenir des coefficients de transfert thermique dans des cas pratiques. Avec des flux de chauffage raisonnables les coefficients de transfert peuvent être trois à quatre fois ceux prédits par les solutions classiques pour des propriétés constantes.

ÜBERLAGERUNG VON ERZWUNGENER UND FREIER KONVEKTION BEI LAMINARER STRÖMUNG IN EINEM HORIZONTALEN ROHR BEI KONSTANTEM WÄRMESTROM

Zusammenfassung—In dieser Arbeit wird der Einfluss der freien Konvektion auf eine laminare Wasserströmung in einem horizontalen zylindrischen Rohr bei konstantem Wärmestron in der Rohrwand untersucht. Für die optische und quantitative Untersuchung wurde ein elektrisch heizbares Glasrohr verwendet. Die Daten wurden mit anderen Daten verglichen. Eine Korrelationsrechnung ergab ein allgemeines Bild des Einflusses der freien Konvektion auf die Nusselt-Zahl. Die endgültigen Korrelationskurven in Abb. 11 werden zur Berechnung des Wärmeübergangskoeffizienten bei praktischen Problemstellungen vorläufig vorgeschlagen.

Bei grösseren Heizflächenbelastungen kann der tatsächliche Wärmeübergangskoeffizient drei- bis viermal so gross werden, als ihn die übliche Rechnung mit konstanten Stoffgrössen liefert.

СОВМЕСТНАЯ ВЫНУЖДЕННАЯ И СВОБОДНАЯ КОНВЕКЦИЯ ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ В ГОРИЗОНТАЛЬНЫХ ТРУБАХ И ОДНОРОДНОМ ТЕПЛОВОМ ПОТОКЕ

Аннотация—В данной работе рассматривается влияние свободной конвекции на ламинарное течение воды в горизонтальных кольцевых трубах, на стенках которых имеет место постоянный поток тепла. С помощью электрически подогреваемой стеклянной трубки выполнены визуальные и количественные исследования. Эти данные вместе с другими данными и корреляциями использовались для получения общей картины влияния свободной конвекции на число Нуссельта. Конечные корреляционные кривые, приведенные на рис. II, рекомендуются для практических расчетов коэффициентов переноса тепла. При умеренных скоростях нагрева коэффициенты переноса тепда могут в 3–4 раза превышать значения, рассчитанные с помощью обычных решений с постоянными свойствами.